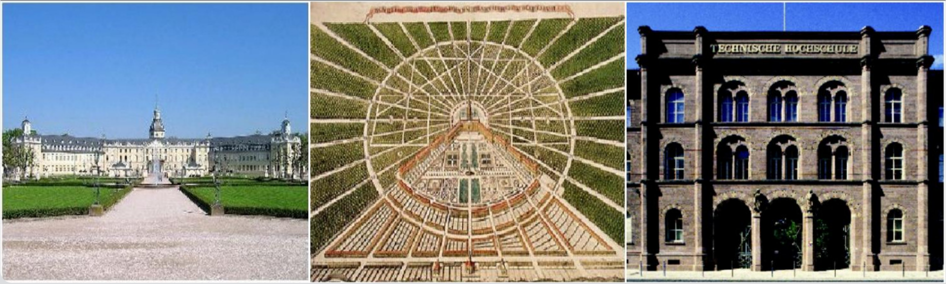
Karlsruhe Institute of Technology

Université franco-allemande
Deutsch-Französische Hochschule

Matti Schneider

Polarization methods: Implementation # 1 $\rightarrow$ # 2

Introduction to FFT-based numerical methods for the homogenization of random materials



Bonus - EM → MMS

$$P^{k+1} = \gamma\, P^k + (1-\gamma)\left[2\mathbb{C}^0 : \bar{\varepsilon} + Y^0 : Z^0(P^k)\right]$$

Bonus - EM $\rightarrow$ MMS

$$P^{k+1} = \gamma\, P^k + (1-\gamma)\left[2\mathbb{C}^0 : \bar{\varepsilon} + Y^0 : Z^0(P^k)\right]$$

- $P^k = \sigma^k + \mathbb{C}^0 : e^k$

Bonus - EM → MMS

Université franco-allemande Deutsch-Französische Hochschule

Karlsruhe Institute of Technology

$$P^{k+1} = \gamma\, P^k + (1-\gamma)\left[2\mathbb{C}^0 : \bar{\varepsilon} + Y^0 : Z^0(P^k)\right]$$

- $P^k = \sigma^k + \mathbb{C}^0 : e^k$
- $\tau^k = \sigma^k - \mathbb{C}^0 : e^k$

Bonus - EM → MMS

$$P^{k+1} = \gamma\, P^k + (1-\gamma)\left[2\mathbb{C}^0 : \bar{\varepsilon} + Y^0 : Z^0(P^k)\right]$$

- $P^k = \sigma^k + \mathbb{C}^0 : e^k$
- $\tau^k = \sigma^k - \mathbb{C}^0 : e^k \equiv Z^0(P^k)$

Bonus - EM → MMS

$$P^{k+1} = \gamma P^k + (1-\gamma)\left[2\mathbb{C}^0 : \bar{\varepsilon} + Y^0 : \tau^k\right]$$

- $P^k = \sigma^k + \mathbb{C}^0 : e^k$
- $\tau^k = \sigma^k - \mathbb{C}^0 : e^k \equiv Z^0(P^k)$

Bonus - EM → MMS

$$P^{k+1} = \gamma\, P^k + (1-\gamma)\left[2\mathbb{C}^0 : \bar{\varepsilon} + Y^0 : \tau^k\right]$$

- $P^k = \sigma^k + \mathbb{C}^0 : e^k$
- $\tau^k = \sigma^k - \mathbb{C}^0 : e^k \equiv Z^0(P^k)$
- $Y^0 = \mathrm{Id} - 2\,\mathbb{C}^0 : \Gamma^0$

Bonus - EM → MMS

$$P^{k+1} = \gamma\, P^k + (1-\gamma)\left[2\mathbb{C}^0 : \bar{\varepsilon} + \left(\mathrm{Id} - 2\,\mathbb{C}^0 : \Gamma^0\right) : \tau^k\right]$$

- $P^k = \sigma^k + \mathbb{C}^0 : e^k$
- $\tau^k = \sigma^k - \mathbb{C}^0 : e^k \equiv Z^0(P^k)$
- $Y^0 = \mathrm{Id} - 2\,\mathbb{C}^0 : \Gamma^0$

Bonus - EM → MMS

$$P^{k+1} = \gamma P^k + (1-\gamma)\left[2\mathbb{C}^0 : \bar{\varepsilon} + \tau^k - 2\,\mathbb{C}^0 : \Gamma^0 : \tau^k\right]$$

- $P^k = \sigma^k + \mathbb{C}^0 : e^k$
- $\tau^k = \sigma^k - \mathbb{C}^0 : e^k \equiv Z^0(P^k)$
- $Y^0 = \mathrm{Id} - 2\,\mathbb{C}^0 : \Gamma^0$

Bonus - EM → MMS

Université franco-allemande Deutsch-Französische Hochschule

KIT Karlsruhe Institute of Technology

$$P^{k+1} = \gamma\, P^k + (1-\gamma)\left[\tau^k + 2\mathbb{C}^0 : \bar{\varepsilon} - 2\,\mathbb{C}^0 : \Gamma^0 : \tau^k\right]$$

- $P^k = \sigma^k + \mathbb{C}^0 : e^k$
- $\tau^k = \sigma^k - \mathbb{C}^0 : e^k \equiv Z^0(P^k)$
- $Y^0 = \mathrm{Id} - 2\,\mathbb{C}^0 : \Gamma^0$

Bonus - EM → MMS

Université franco-allemande Deutsch-Französische Hochschule

Karlsruhe Institute of Technology

$$P^{k+1} = \gamma P^k + (1-\gamma)\left[\tau^k + 2\mathbb{C}^0 : \left(\bar{\varepsilon} - \Gamma^0 : \tau^k\right)\right]$$

- $P^k = \sigma^k + \mathbb{C}^0 : e^k$
- $\tau^k = \sigma^k - \mathbb{C}^0 : e^k \equiv Z^0(P^k)$
- $Y^0 = \mathrm{Id} - 2\,\mathbb{C}^0 : \Gamma^0$

Bonus - EM → MMS

Université franco-allemande Deutsch-Französische Hochschule

Karlsruhe Institute of Technology

$$P^{k+1} = \gamma P^k + (1-\gamma)\left[\tau^k + 2\mathbb{C}^0 : \left(\bar{\varepsilon} - \Gamma^0 : \tau^k\right)\right]$$

- $P^k = \sigma^k + \mathbb{C}^0 : e^k$
- $\tau^k = \sigma^k - \mathbb{C}^0 : e^k \equiv Z^0(P^k)$
- $Y^0 = \mathrm{Id} - 2\,\mathbb{C}^0 : \Gamma^0$
- $\varepsilon^{k+1/2} = \bar{\varepsilon} - \Gamma^0 : \tau^k$

Bonus - EM → MMS

$$P^{k+1} = \gamma\, P^k + (1-\gamma)\left[\tau^k + 2\mathbb{C}^0 : \varepsilon^{k+1/2}\right]$$

- $P^k = \sigma^k + \mathbb{C}^0 : e^k$
- $\tau^k = \sigma^k - \mathbb{C}^0 : e^k \equiv Z^0(P^k)$
- $Y^0 = \mathrm{Id} - 2\,\mathbb{C}^0 : \Gamma^0$
- $\varepsilon^{k+1/2} = \bar{\varepsilon} - \Gamma^0 : \tau^k$

Bonus - EM → MMS

$$P^{k+1} = \gamma\, P^k + (1-\gamma)\, \tau^k + 2(1-\gamma)\, \mathbb{C}^0 : \varepsilon^{k+1/2}$$

- $P^k = \sigma^k + \mathbb{C}^0 : e^k$
- $\tau^k = \sigma^k - \mathbb{C}^0 : e^k \equiv Z^0(P^k)$
- $Y^0 = \mathrm{Id} - 2\, \mathbb{C}^0 : \Gamma^0$
- $\varepsilon^{k+1/2} = \bar{\varepsilon} - \Gamma^0 : \tau^k$

Bonus - EM → MMS

$$P^{k+1} = \gamma\,(\sigma^k + \mathbb{C}^0 : e^k) + (1-\gamma)\,\tau^k + 2(1-\gamma)\,\mathbb{C}^0 : \varepsilon^{k+1/2}$$

- $P^k = \sigma^k + \mathbb{C}^0 : e^k$
- $\tau^k = \sigma^k - \mathbb{C}^0 : e^k \equiv Z^0(P^k)$
- $Y^0 = \mathrm{Id} - 2\,\mathbb{C}^0 : \Gamma^0$
- $\varepsilon^{k+1/2} = \bar{\varepsilon} - \Gamma^0 : \tau^k$

Bonus - EM → MMS

$$P^{k+1} = \gamma\,(\sigma^k + \mathbb{C}^0 : e^k) + (1-\gamma)\,(\sigma^k - \mathbb{C}^0 : e^k) + 2(1-\gamma)\,\mathbb{C}^0 : \varepsilon^{k+1/2}$$

- $P^k = \sigma^k + \mathbb{C}^0 : e^k$
- $\tau^k = \sigma^k - \mathbb{C}^0 : e^k \equiv Z^0(P^k)$
- $Y^0 = \mathrm{Id} - 2\,\mathbb{C}^0 : \Gamma^0$
- $\varepsilon^{k+1/2} = \bar{\varepsilon} - \Gamma^0 : \tau^k$

Bonus - EM → MMS

$$P^{k+1} = \gamma\,\sigma^k + \gamma\,\mathbb{C}^0 : e^k + (1-\gamma)\,\sigma^k - (1-\gamma)\,\mathbb{C}^0 : e^k + 2(1-\gamma)\,\mathbb{C}^0 : \varepsilon^{k+1/2}$$

- $P^k = \sigma^k + \mathbb{C}^0 : e^k$
- $\tau^k = \sigma^k - \mathbb{C}^0 : e^k \equiv Z^0(P^k)$
- $Y^0 = \mathrm{Id} - 2\,\mathbb{C}^0 : \Gamma^0$
- $\varepsilon^{k+1/2} = \bar{\varepsilon} - \Gamma^0 : \tau^k$

Bonus - EM → MMS

Université franco-allemande Deutsch-Französische Hochschule

Karlsruhe Institute of Technology

$$P^{k+1} = \gamma\,\sigma^k + (1-\gamma)\,\sigma^k + \gamma\,\mathbb{C}^0 : e^k - (1-\gamma)\,\mathbb{C}^0 : e^k + 2(1-\gamma)\,\mathbb{C}^0 : \varepsilon^{k+1/2}$$

- $P^k = \sigma^k + \mathbb{C}^0 : e^k$
- $\tau^k = \sigma^k - \mathbb{C}^0 : e^k \equiv Z^0(P^k)$
- $Y^0 = \mathrm{Id} - 2\,\mathbb{C}^0 : \Gamma^0$
- $\varepsilon^{k+1/2} = \bar{\varepsilon} - \Gamma^0 : \tau^k$

Bonus - EM → MMS

$$P^{k+1} = \sigma^k + \gamma\, \mathbb{C}^0 : e^k - (1-\gamma)\, \mathbb{C}^0 : e^k + 2(1-\gamma)\, \mathbb{C}^0 : \varepsilon^{k+1/2}$$

- $P^k = \sigma^k + \mathbb{C}^0 : e^k$
- $\tau^k = \sigma^k - \mathbb{C}^0 : e^k \equiv Z^0(P^k)$
- $Y^0 = \mathrm{Id} - 2\, \mathbb{C}^0 : \Gamma^0$
- $\varepsilon^{k+1/2} = \bar{\varepsilon} - \Gamma^0 : \tau^k$

Bonus - EM → MMS

$$P^{k+1} = \sigma^k - (1-2\gamma)\,\mathbb{C}^0 : e^k + 2(1-\gamma)\,\mathbb{C}^0 : \varepsilon^{k+1/2}$$

- $P^k = \sigma^k + \mathbb{C}^0 : e^k$
- $\tau^k = \sigma^k - \mathbb{C}^0 : e^k \equiv Z^0(P^k)$
- $Y^0 = \mathrm{Id} - 2\,\mathbb{C}^0 : \Gamma^0$
- $\varepsilon^{k+1/2} = \bar{\varepsilon} - \Gamma^0 : \tau^k$

Bonus - EM → MMS

Université franco-allemande Deutsch-Französische Hochschule

KIT Karlsruhe Institute of Technology

$$P^{k+1} = \sigma^k + \mathbb{C}^0 : \left[-(1-2\gamma)\, e^k + 2(1-\gamma)\, \varepsilon^{k+1/2}\right]$$

- $P^k = \sigma^k + \mathbb{C}^0 : e^k$
- $\tau^k = \sigma^k - \mathbb{C}^0 : e^k \equiv Z^0(P^k)$
- $Y^0 = \mathrm{Id} - 2\,\mathbb{C}^0 : \Gamma^0$
- $\varepsilon^{k+1/2} = \bar{\varepsilon} - \Gamma^0 : \tau^k$

Bonus - EM → MMS

$$P^{k+1} = \sigma^k + \mathbb{C}^0 : \left[-(1-2\gamma)\, e^k + 2(1-\gamma)\, \varepsilon^{k+1/2} \right]$$

- $P^k = \sigma^k + \mathbb{C}^0 : e^k$
- $\tau^k = \sigma^k - \mathbb{C}^0 : e^k \equiv Z^0(P^k)$
- $Y^0 = \mathrm{Id} - 2\, \mathbb{C}^0 : \Gamma^0$
- $\varepsilon^{k+1/2} = \bar{\varepsilon} - \Gamma^0 : \tau^k$
- $\varepsilon^k = (1-2\gamma)\, e^k + 2(1-\gamma)\, \varepsilon^{k+1/2}$

Bonus - EM → MMS

$$P^{k+1} = \sigma^k + \mathbb{C}^0 : \varepsilon^k$$

- $P^k = \sigma^k + \mathbb{C}^0 : e^k$
- $\tau^k = \sigma^k - \mathbb{C}^0 : e^k \equiv Z^0(P^k)$
- $Y^0 = \mathrm{Id} - 2\, \mathbb{C}^0 : \Gamma^0$
- $\varepsilon^{k+1/2} = \bar{\varepsilon} - \Gamma^0 : \tau^k$
- $\varepsilon^k = (1 - 2\gamma)\, e^k + 2(1 - \gamma)\, \varepsilon^{k+1/2}$

Bonus - EM → MMS

$$\sigma^{k+1} + \mathbb{C}^0 : e^{k+1} = \sigma^k + \mathbb{C}^0 : \varepsilon^k$$

- $P^k = \sigma^k + \mathbb{C}^0 : e^k$
- $\tau^k = \sigma^k - \mathbb{C}^0 : e^k \equiv Z^0(P^k)$
- $Y^0 = \mathrm{Id} - 2\, \mathbb{C}^0 : \Gamma^0$
- $\varepsilon^{k+1/2} = \bar{\varepsilon} - \Gamma^0 : \tau^k$
- $\varepsilon^k = (1 - 2\gamma)\, e^k + 2(1 - \gamma)\, \varepsilon^{k+1/2}$

Bonus - EM → MMS

$$\sigma^{k+1} + \mathbb{C}^0 : e^{k+1} = \sigma^k + \mathbb{C}^0 : \varepsilon^k \quad \text{and} \quad \sigma^{k+1} + \mathbb{C}^0 : e^{k+1} = \sigma^k + \mathbb{C}^0 : \varepsilon^k$$

- $P^k = \sigma^k + \mathbb{C}^0 : e^k$
- $\tau^k = \sigma^k - \mathbb{C}^0 : e^k \equiv Z^0(P^k)$
- $Y^0 = \mathrm{Id} - 2\,\mathbb{C}^0 : \Gamma^0$
- $\varepsilon^{k+1/2} = \bar{\varepsilon} - \Gamma^0 : \tau^k$
- $\varepsilon^k = (1 - 2\gamma)\, e^k + 2(1-\gamma)\, \varepsilon^{k+1/2}$

Bonus - EM → MMS

$$\sigma^{k+1} + \mathbb{C}^0 : e^{k+1} = \sigma^k + \mathbb{C}^0 : \varepsilon^k \quad \text{and} \quad \sigma^{k+1} + \mathbb{C}^0 : e^{k+1} = \sigma^k + \mathbb{C}^0 : \varepsilon^k$$

- $P^k = \sigma^k + \mathbb{C}^0 : e^k$ with $\sigma^k = \mathsf{S}(\cdot, e^k)$
- $\tau^k = \sigma^k - \mathbb{C}^0 : e^k \equiv Z^0(P^k)$
- $Y^0 = \mathrm{Id} - 2\,\mathbb{C}^0 : \Gamma^0$
- $\varepsilon^{k+1/2} = \bar{\varepsilon} - \Gamma^0 : \tau^k$
- $\varepsilon^k = (1 - 2\gamma)\, e^k + 2(1 - \gamma)\, \varepsilon^{k+1/2}$

Bonus - EM → MMS

$$\mathsf{S}(\cdot, e^{k+1}) + \mathbb{C}^0 : e^{k+1} = \sigma^k + \mathbb{C}^0 : \varepsilon^k \quad \text{and} \quad \sigma^{k+1} + \mathbb{C}^0 : e^{k+1} = \sigma^k + \mathbb{C}^0 : \varepsilon^k$$

- $P^k = \sigma^k + \mathbb{C}^0 : e^k \quad \text{with} \quad \sigma^k = \mathsf{S}(\cdot, e^k)$
- $\tau^k = \sigma^k - \mathbb{C}^0 : e^k \equiv Z^0(P^k)$
- $Y^0 = \text{Id} - 2\,\mathbb{C}^0 : \Gamma^0$
- $\varepsilon^{k+1/2} = \bar{\varepsilon} - \Gamma^0 : \tau^k$
- $\varepsilon^k = (1 - 2\gamma)\, e^k + 2(1 - \gamma)\, \varepsilon^{k+1/2}$

Bonus - EM → MMS

Université franco-allemande Deutsch-Französische Hochschule

Karlsruhe Institute of Technology

$$\mathsf{S}(\cdot, e^{k+1}) + \mathbb{C}^0 : e^{k+1} = \sigma^k + \mathbb{C}^0 : \varepsilon^k \quad \text{and} \quad \sigma^{k+1} = \sigma^k + \mathbb{C}^0 : (\varepsilon^k - e^{k+1})$$

- $P^k = \sigma^k + \mathbb{C}^0 : e^k \quad \text{with} \quad \sigma^k = \mathsf{S}(\cdot, e^k)$
- $\tau^k = \sigma^k - \mathbb{C}^0 : e^k \equiv Z^0(P^k)$
- $Y^0 = \mathrm{Id} - 2\, \mathbb{C}^0 : \Gamma^0$
- $\varepsilon^{k+1/2} = \bar{\varepsilon} - \Gamma^0 : \tau^k$
- $\varepsilon^k = (1 - 2\gamma)\, e^k + 2(1 - \gamma)\, \varepsilon^{k+1/2}$

Bonus - EM → MMS

$$S(\cdot, e^{k+1}) + \mathbb{C}^0 : e^{k+1} = \sigma^k + \mathbb{C}^0 : \varepsilon^k \quad \text{and} \quad \sigma^{k+1} = \sigma^k + \mathbb{C}^0 : (\varepsilon^k - e^{k+1})$$

- $\varepsilon^{k+1/2} = \bar{\varepsilon} - \Gamma^0 : (\sigma^k - \mathbb{C}^0 : e^k)$
- $\varepsilon^k = (1 - 2\gamma)\, e^k + 2(1 - \gamma)\, \varepsilon^{k+1/2}$